Module 3: Relations and Functions

Problems on Cartesian Product

1. Define cartesian product of two sets
2. Define power set. For any sets A, B, C≤U, Prove that
   (a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
   (b) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
   (c) $(A \cup B) \times C = (A \times C) \cup (B \times C)$
   (d) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

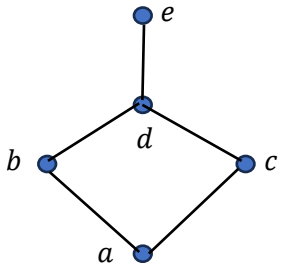
Problems on Relations

3. If A is a set with $'m'$ elements and B is a set with $'n'$ elements, find the number of relations from $A\ to\ B$.
4. Let A and B be finite sets with $|B| = 3$. If there are 4096 relations from $A\ to\ B$, what is $|A|$?
5. Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 5\}$. Determine the following:
   (a) $|A \times B|$
   (b) Number of relations from $A\ to\ B$.
   (c) Number of binary relations on A.
   (d) Number of relations from $A\ to\ B$ that contain (1, 2) and (1, 5).
   (e) Number of relations from A, B that contains exactly five ordered pairs.
   (f) Number of binary relations on A that contains at least seven ordered pairs.
6. Let $A = \{1, 2, 3, 4, 6\}$ and R be the relation on A defined by $(a, b) \in R$ if and only if a is multiple of b. Write down R as a set of ordered pairs.

Problems on properties of Relations

7. Let N be the set of Natural numbers. Let a relation R be defined by R={(a, b) / a ∈ N, b ∈ N, a – b is divisible by 5}. Prove that R is an equivalence relation.
8. Let $A = \{1, 2, 3, 4\}$ and R be the relation on A defined by xRy if and only if $x < y$. Write down R as a set of ordered pairs. Write the relation matrix and draw the diagraph. List out the in degrees and out degrees of every vertex.
9. Let $A = \{1, 2, 3, 4, 6\}$ and 'R' be the relation on 'A' defined by aRb if and only if "a is multiple of b" represent the relation 'R' as a matrix $M(R)$, draw the diagraph and relation R. List out its indegree and out degree.
10. If R is a relation on $A = \{1, 2, 3, 4\}$ defined by xRy if "x divides y". Prove that (A, R) is a POSET. Draw its Hasse diagram.
11. Draw the Hasse diagram representing the positive divisor of 36.
12. Draw the Hasse diagram representing the positive divisor of 72.
13. Let $A = \{1, 2, 3, 6, 9, 12, 18\}$ and define R on A by xRy iff 'x divides y'. Prove that (A, R) is a POSET. Draw the Hasse diagram for (A, R).

14. For $A = \{a, b, c, d, e\}$, the Hasse diagram for the POSET (A, R) is as shown below:
    (i) Determine the relation matrix for R
    (ii) Construct the digraph for R



15. Let $A = \{a, b, c, d\}$ and $B = \{1, 2, 3, 4, 5\}$. Find the number of one-one functions and onto functions from (i) A to B (ii) B to A.
16. Let $A = \{1, 2, 3, 4, 5\}$. Define a relation R on $A \times A$ by $(x_1, y_1)R(x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$.
    (i) Verify that R is an equivalence relation.
    (ii) Determine the equivalence class of [(2, 4)].

Problems on Pigeon hole principle

17. State pigeon hole principle. Prove that in any set of 29 persons, at least 5 persons have been born on the same day of the week.
18. State pigeon hole principle. Prove that if any number from 1 to 8 are chosen then two of them will have their sum as 9.
19. State pigeon hole principle. Using pigeon hole principle find the minimum number of persons chosen so that at least 5 of them will have their birthday in the same month.
20. How many persons must be chosen in order that at least five of them will have birth days in the same calendar month?
21. State pigeon hole principle. Show that if n+1 numbers are chosen from 1 to 2n then at least one pair add to 2n+1.
22. State pigeon hole principle. Prove that if 30 dictionaries in a library contains a total of 61,327 pages then at least one of dictionaries must have at least 2045 pages.

Problems on Functions

23. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$,
    (i) How many functions are there from A to B?
    (ii) How many of these are one to one function?
    (iii) How many functions are there from B to A?
    (iv) How many functions are these are onto function?
24. Let $A = \{a, b, c, d\}$ and $B = \{1, 2, 3, 4, 5\}$. Find the number of one-one functions and onto functions from (i) A to B (ii) B to A.
25. Let A and B be finite sets. IF there are 60 one-to-one functions from A to B and $|A| = 3$, what is $|B|$?

26. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{w, x, y, z\}$. Find the number of onto functions from A to B.
27. Consider the functions f and g from R to R defined by $f(x) = 2x + 5$ and $g(x) = \frac{(x-5)}{2}$. Prove that g is inverse of f.
28. Let f and g be functions from R to R defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$ if $(gof)(x) = 9x^2 - 9x + 3$, determine a and b.
29. Let $A = B = C = R$, and $f: A \to B$ and $g: B \to C$ be defined by $f(a) = 2a + 1$, $g(b) = \frac{1}{3}b$, $\forall a \in A, \forall b \in B$. Compute gof and show that gof is invertible. What if $(gof)^{-1}$?
30. Let $f: R \to R$ be defined by $f(x) = \begin{cases} 3x - 5 & if \ \ x > 0 \\ 1 - 3x & if \ \ x \leq 0 \end{cases}$. Find $f(1), f(-1), \ f\left(\frac{5}{3}\right), f\left(-\frac{5}{3}\right)$, $f^{-1}(0), \ f^{-1}(1), \ f^{-1}(-1), \ f^{-1}(3), f^{-1}(-3), f^{-1}(-6),$ and $f^{-1}([-5, 5]), f^{-1}([-6, 5])$.